The Pathmox approach for PLS path modeling: discovering which constructs differentiate segments

Abstract

The problem of heterogeneity represents a very important issue in the decision-making process. Furthermore, it has become common practice in the context of marketing research to assume that different population parameters are possible depending on socio-demographic and psycho-demographic variables such as age, gender and social-status. In recent decades, numerous approaches have been proposed with the aim of involving heterogeneity in the parameter estimation procedures. In partial least squares path modeling (PLS-PM), the common practice consists of achieving a global measurement of the differences arising from heterogeneity. This leaves the analyst with the important task of detecting, a posteriori, which are the causal relationships (i.e., path coefficients) that produce changes in the model. This is the case in Pathmox analysis, which solves the heterogeneity problem by building a binary tree to detect those segments of population that cause the heterogeneity. In this article, we propose extending the same Pathmox methodology in order to asses which particular endogenous equation of the structural model and which path coefficients are responsible of the difference.

Keywords: Heterogeneity, Partial least squares path modeling, Segmentation, Pathmox, Models comparison, Fisher's F

1 Introduction

It is a known fact that consumer needs evolve very quickly and that good marketing strategies are closely linked with providers' capacity "to tailor" a "custom-made suit" for their own clients. This modern point of view has given heterogeneity analysis top priority in the field of research. Not surprisingly, many techniques in the context of partial least squares path modeling (PLS-PM) have been proposed with the principal aim of involving heterogeneity in parameter estimation procedures ([1], [2] and [3]).

In general, the principal approaches focus on identifying the significant presence of any heterogeneity, leaving the analyst with the task of researching the causal relationship that produces the changes in the model. This task can be difficult, depending on the complexity of the model. From this perspective, it is fundamental to detect differences among segments of customers while, at the same time, to identify the reasons behind such differences[1].

The Pathmox algorithm [4] detects significant partitions in the presence of heterogeneity. However, it does not provide any information about the endogenous equations and the path coefficients that cause differences between two subgroups.

The objective of this paper is to present two tests that overcome this limitation: the F-block test and the F-coefficient test.

In Sections 2 and 3, we provide a literary review of the principal techniques and a brief description of the Pathmox algorithm. In Section 4, we present an application to illustrate and justify the proposed extensions. In Section 5, we discuss our contribution: the F-block test and the F-coefficient test. In Section 6, we show a simulation study with artificial data in order to evaluate the sensitivity of the tests. In Section 7, we present the results from applying the two

[1] We point out that we only consider heterogeneity resulting from differences in the path coefficients of the structural models.

tests in the Pathmox analysis. The paper closes with conclusions on the suitability of the proposed approach (Sections 8 and 9).

2 Literary review

To the extent of our knowledge, the techniques that identify significant coefficients responsible for differences arising from the presence of heterogeneity, are related to PLS-based multi-group analysis (MGA). In MGA, a population parameter, b, is hypothesized to differ for two subpopulations: $b_1 \neq b_2$. This focus consider heterogeneity (i.e., two sub-models are estimated by considering segmentation variables) by testing differences in the PLS-PM path coefficients of the two models.

The primary approach for group comparisons is a t-test, as described by [5]. This test is used in the resampling parametric approach proposed by [6]. The procedure consists of separating the data into groups (i.e., segments) and running bootstrap resamplings for each group. Path coefficients are calculated in each resampling, and the standard error estimates are treated in a parametric sense via t-test. As stated by the author, this approach works reasonably well if the two samples are not too non-normal and/or the two variances are not too different from one another; the principal limitation is bound to the use of a parametric test in a PLS-PM context, where no distributional assumptions are made.

In 2003, [7] and [8] proposed the resampling non-parametric approach as an alternative for overcoming the problem of the parametric assumption of the t-test. This approach seeks to scale the observed differences between groups by comparing these differences to those that exist between groups that have been randomly assembled from the data. The procedure starts by calculating PLS-PM models separately for each group (*population parameters*); it then permutes the data and, for each obtained group, it fits the parameter estimates (*permutation parameter*). Finally, it computes the differences in the *permutation parameter* estimates and tests the null hypothesis that the *population parameters* are equal across the two groups.

A more recent proposal was presented by [9]. This approach is based on bootstrapping, and it consists of obtaining the empirical cumulative distribution of the parameters under consideration. First, the subsamples to be compared are exposed to separate bootstrap analyses, and the bootstrap outcomes serve as a basis for the hypothesis tests of group differences. Instead of relying on distributional assumptions, the new approach estimates the probability that one path coefficient exceeds the other. However, as remarked by [9], this approach only tests the one-sided hypotheses.

As an answer to the deficiencies of prior methods, [3] proposed the confidence set approach, which builds conceptually on [5]'s parametric test. [5]'s approach is a modified version of the two independent samples t-test, which accounts for the fact that the standard deviation is obtained through bootstrapping. In accordance with this test, researchers can directly compare the group-specific bootstrap confidence intervals, regardless of whether the data are normally distributed or not. If the parameter estimates fall within the corresponding confidence interval, it can be assumed that there are no significant differences between the group-specific path coefficients.

A comparison of these approaches is made by [3]. As he points out, the resampling parametric approach [6] is the most liberal of the procedures, as it generally yields higher t-values when compared to the permutation test of the resampling non-parametric approach ([7], [8]). By not relying on distributional assumptions, the resampling non-parametric approach overcomes a key disadvantage of the resampling parametric approach and thus, fits with the characteristics of the PLS path modeling method. However, the permutation-based approach requires group-specific sample sizes to be fairly similar, which is a considerable limitation. The procedures of [9] and [3] appear to be very conservative, as they indicate fewer significant differences vis-à-vis alternative multi-group comparison tests.

Without entering into the debate about what is the best technique and which differences are the most significant, we can consider more generally the way in which these authors analyze heterogeneity. In our opinion, the multi-group approach represents a good option when we know the effect we wish to prove is significant. In practice, this implies that the number of groups is known a priori and there is a limited number of segmentation variables: two or a maximum of three. It is also a viable option when there is previous knowledge regarding which available factors produce significant differences in the analysis results. However, very often there are more than two segmentation variables, and we do not have previous knowledge about the factors defining the heterogeneity.

In such situations, fitting for each cross-level of the segmentation variables in a different model will make it rather difficult for the analyst to find the significant partitions.

3 The Pathmox approach

The Pathmox approach [4] represents a viable alternative for working with survey data for which there are numerous segmentation variables and there is no previous knowledge about the factors. This algorithm is proposed with the aim of developing a new segmentation approach for observed heterogeneity in PLS-PM. This technique adapts the principles of the binary segmentation process by producing a segmentation tree with different path models in each of the obtained nodes. The goal of Pathmox is not prediction but rather identification; i.e., its purpose is to detect different path models present in the data. Thus, Pathmox identifies the set of splits (based on the segmentation variables) with superior discriminating capacity, in the sense that it separates PLS-PM models as much as possible. Here, we have adapted a split criterion based on Fisher's F for testing the equality of regression models ([10] and [11]). This allows us to decide whether two structural models calibrated from two different segments (successors of a node) can be considered as different. We call this the F-global test. To identify the existence of different path models, the technique performs a procedure that can be summarized in the following algorithm:

Algorithm 1 Pathmox Algorithm

Step 1. Start with the global PLS path model at the root node
Step 2. Establish a set of admissible partitions for each segmentation variable in each node of the tree
Step 3. Detect the best partition by:

- **3.1.** Forming all binary partitions in all segmentation variables
- **3.2.** Applying the F-test, calculating a p-value for each partition
- **3.3.** Sorting the p-values in descending order
- **3.4.** Choosing, as the best partition ,the one associated with the lowest p-value

Step 4. ***If*** (stop criteria[1] = **false**) ***then*** repeat **step 3**

1. Possible stop criteria:
a. The p-values in the F-global test are not significant
b. The number of individuals in the group falls below a fixed level
c. The maximum level of the tree depth is attained

The algorithm starts by estimating the global PLS path model (over all the observations) at the root node. Then, the explanatory external variables are used to produce all possible binary splits. Among all the possible splits, the best one is selected by means of an F-test for comparing inner models. This process is recursively applied for each child node. The stop criterion is based on the significance level of the p-value associated with the F statistic. Additionally, two stop parameters are also considered: (1) the number of individuals in a node and (2) the depth of the tree's growing level.

Before discussing the extension of the Pathmox approach in Section 5, we illustrate how Pathmox works in Section 4.

4 Pathmox application: estimating the customer satisfaction of a Spanish bank

To illustrate how Pathmox deals with the problem of heterogeneity, we use an example based on customer satisfaction measurement, which is the most typical application of PLS-PM in marketing research ([12], [13]).

The objective is to elucidate the presence of heterogeneity and establish its potential causes. In particular, our aim is to investigate whether there are any differences in *Customer Satisfaction* and *Loyalty* for one of Spain's leading providers of retail financial services.

For this approach, we need to differentiate between two sets of variables: one is made up of the manifest variables that will define the PLS-PM model and the other consists of the segmentation variables. Our case study includes manifest variables measured on an 11-point ordinal scale, ranging from very satisfied (10) to very dissatisfied (0) (see Table 1) and 4 segmentation variables: Gender, Age, Education Level and Occupation (see Table 2), all of them measured from 420 individuals.

The model includes six constructs: *Image*, *Expectations*, *Perceived Quality*, *Perceived Value*, *Customer Satisfaction*, and *Loyalty*. It is designed to measure the cause-effect relationships between the antecedents and consequences of *Customer Satisfaction* and *Loyalty*. The antecedents of *Customer Satisfaction* are *Image*, *Expectations*, *Perceived Quality*, and *Perceived Value*, while those of *Loyalty* are *Customer Satisfaction* and *Image*. The definitions of the theoretical constructs of the model are given in Table 3. We assume that all latent variables are reflective.

4.1 Global PLS-PM analysis

We begin with the calculation of the global PLS model for all customers (Figure 1). In order to simplify the interpretation, we merely present the results of *Customer Satisfaction* and *Loyalty*. As indicated by [14] we analyze (in order): unidimensionality, reliability (for validating the outer model and the path coefficient estimates), their significance, and the predictability of the model (for understanding the causal relationships that determine *Customer Satisfaction* and *Loyalty*). All the results were obtained with the R package "plspm" [15]. The results were:

1. The *unidimensionality* of the reflective latent constructs (Table 4). All Crombach's α are higher than 0.82.

2. *Reliability*, as measured by the average variance extracted for each construct in respect to its indicators (Table 5). In all cases, it is greater than 0.60, meaning indicator coherence in measuring the construct.

3. The *estimated inner model* (Figure 2). We can see that the main (direct) drivers for *Customer Satisfaction* are *Perceived Value* and *Perceived Quality*, whereas *Customer Satisfaction* is the main antecedent for *Loyalty*.

4. The *significance of the path coefficients*. We assess this by looking at the bootstrap confidence intervals of path coefficients (Table 6), of which all are significant.

5. *Predictability of the model* (Table 5), with an R^2 of 0.647 and 0.653 for the *Customer Satisfaction* and *Loyalty* constructs, respectively. These are normal values in customer satisfaction studies.

4.2 Pathmox analysis

Despite the reduced sample size of the study, it seems that the obtained model is good enough to draw conclusions about how *Customer Satisfaction* and *Loyalty* are formed among the banks' customers. However, the question remains about whether the effects we have detected are valid or if they are artificial averages of underlying subpopulations. To answer that question and discover potential sources of heterogeneity, we performed a Pathmox analysis[2] using the segmentation variables of Table 2.

[2]The Pathmox analysis was performed by using the "genpathmox" R package [16].

The tree we are interested in has a moderate final number of segments, which can be interpreted and made operational for marketing purposes. In this case, we have limited the tree by imposing: a maximum depth of one level for the sake of interpretability; a minimum admissible size of 10% for splitting nodes; and a threshold significance that is set at 0.05 for the split criterion.

The obtained Pathmox tree is shown in Figure 3. It is a tree with a total number of two terminal nodes. The root node corresponds to the previous global model that was calculated over the entire sample size. The split results from the segmentation variable Education, which has a corresponding F statistic p-value of 0.001. This split divides the 420 customers of the root node into two subsets. One subset is made up of 391 "non-graduate" customers (node 2). The other subset consists of 29 "graduate" customers (node 3).

In Figure 4, we present the PLS-PM models corresponding to the two segments. To better appreciate the differences, we marked in red the arrows of coefficients that are most significantly different in both segments. As we can see, there are two path coefficients that have very different effects in the two models: *Perceived Quality* on *Customer Satisfaction*, *Perceived Value* on *Customer Satisfaction*. In the model of "non-graduate" customers (node 2), we can see that *Perceived Quality* presents a positive effect on *Customer Satisfaction* (path coefficient: 0.294) and *Perceived Value* a positive effect on *Customer Satisfaction* (path coefficient: 0.375); whereas in the model of "graduate" customers (node 3), the effect of *Perceived Quality* on *Customer Satisfaction* is negative (path coefficient: -0.256) and the effect of *Perceived Value* on *Customer Satisfaction* is strongly positive (path coefficient: 0.764).

In Table 7, we show the direct, indirect and total effects among the constructs, where we corroborate the importance of *Image*, *Expectations*, *Perceived Quality*, and *Perceived Value*, for the "non-graduate" customers in defining *Customer Satisfaction* and *Loyalty*. On the other hand, *Perceived Quality* appears not to be relevant for the "graduate" customers.

Finally, we also provide validation of the path coefficients (see Table 8) by calculating the bootstrap intervals for each path coefficient of the model, which is useful for understanding whether the identified differences are significant. Starting with the model of "non-graduate" customers (node 2) we observe that all path coefficients are significant, except for *Expectations* on *Customer Satisfaction*. Looking at node 3 (model of "graduate" customers), we have only two path coefficients that are significant: *Perceived Value* on *Customer Satisfaction* and *Customer Satisfaction* on *Loyalty*, meaning that this target considers only *Perceived Value* as significant in defining their *Customer Satisfaction*.

5 Extending the Pathmox analysis

The F-test used as a split criterion in Pathmox is a global criterion: it assess whether all the path coefficients of all the equations in the structural model are equal when comparing two groups, but it does not indicate which particular structural equation or which path coefficients are responsible for the difference. We need an a posteriori comparison of the child nodes to understand the reason (i.e., causal relationships) of such a difference. For instance, when Pathmox detects a difference between the two groups (the "graduate" customers and the "non-graduate" customers), we do not know which of the six structural equations (one for each endogenous latent variable) is responsible for the detected difference. In the case where we have a significant difference in one structural equation (for instance in *Customer Satisfaction*), we do not know which path coefficient is responsible (*Image*, *Expectation*, *Perceived Value*, or *Perceived Quality*). In order to distinguish the equation and the path coefficients that are responsible for the split, we introduced the F-block test and the F-coefficient test.

5.1 F-block test

To detect which regression (i.e., structural equation) is responsible for the global difference, we extended the F-global test. We will call the statistic of this comparison the F-block (or block-test). Let us consider a structural model (Figure 5) with two endogenous variables, η_1 and η_2, and one exogenous variable ξ_1. The structural equations for both endogenous constructs are:

$$\eta_1 = \beta_1\xi_1 + \zeta_1 \tag{1}$$
$$\eta_2 = \beta_2\xi_1 + \beta_3\eta_1 + \zeta_2 \tag{2}$$

The disturbance terms ζ_1 and ζ_2 are assumed to be normally distributed with zero mean and finite and equal variance: that is, $E(\zeta_1) = E(\zeta_2) = 0$ and $Var(\zeta_1) = Var(\zeta_2) = \sigma^2$ I. It is also assumed that $Cov(\zeta_1, \zeta_2) = 0$. We define the following matrices:

$X_1 = [\xi_1]$	a column matrix with the explicative latent variables of η_1
$B_1 = [\beta_1]$	a vector of path coefficients for the regression of η_1
$X_2 = [\xi_1, \eta_1]$	a column matrix with the explicative latent variables of η_2
$B_2 = [\beta_2, \beta_3]$	a vector of path coefficients for the regression of η_2

Then, the structural equations are expressed as:

$$\eta_1 = X_1 B_1 + \zeta_1 \tag{3}$$
$$\eta_2 = X_2 B_2 + \zeta_2 \tag{4}$$

We assume that the parent node is divided into two child nodes or segments, one containing n_A elements and the other containing n_B observations. For each segment, we compute its own structural model:

$$SegmentA : \eta_1^A = X_1^A B_1^A + \zeta_1^A \quad \text{and} \quad \eta_2^A = X_2^A B_2^A + \zeta_2^A \tag{5}$$
$$SegmentB : \eta_1^B = X_1^B B_1^B + \zeta_1^B \quad \text{and} \quad \eta_2^B = X_2^B B_2^B + \zeta_2^B \tag{6}$$

with $\zeta_1^A \sim N(0, \sigma^2 I)$ and $\zeta_2^A \sim N(0, \sigma^2 I), \zeta_1^B \sim N(0, \sigma^2 I)$ and $\zeta_2^B \sim N(0, \sigma^2 I)$.

Let us assume that the F-global test gives a significant p-value. We wish to investigate which equation (η_1 or η_2) is responsible for the difference. For the sake of simplicity, we wish to validate whether the first equation is equal in both segments while letting equation two vary freely. In this case the null hypothesis, H_0, is that the equation shown in (1) is equal for segments

A and B, while the alternative hypothesis, H_1, is that all equations are different. Following the rationale of [4], both hypotheses can be written as follows:

$$H_0: \underset{[2n,1]}{\begin{bmatrix}\eta_1^A\\ \eta_2^A\\ \eta_1^B\\ \eta_2^B\end{bmatrix}} = \underset{[2n,p_1+2p_2]}{\begin{bmatrix}X_1^A & 0 & 0\\ 0 & X_2^A & 0\\ X_1^B & 0 & 0\\ 0 & 0 & X_2^B\end{bmatrix}} \underset{[p_1+2p_2,1]}{\begin{bmatrix}\beta_1\\ \beta_2^A\\ \beta_2^B\end{bmatrix}} + \underset{[2n,1]}{\begin{bmatrix}\zeta_1^A\\ \zeta_2^A\\ \zeta_1^B\\ \zeta_2^B\end{bmatrix}} \tag{7}$$

$$H_1: \underset{[2n,1]}{\begin{bmatrix}\eta_1^A\\ \eta_2^A\\ \eta_1^B\\ \eta_2^B\end{bmatrix}} = \underset{[2n,2p_1+2p_2]}{\begin{bmatrix}X_1^A & 0 & 0 & 0\\ 0 & X_2^A & 0 & 0\\ 0 & 0 & X_1^B & 0\\ 0 & 0 & 0 & X_2^B\end{bmatrix}} \underset{[2p_1+2p_2,1]}{\begin{bmatrix}\beta_1^A\\ \beta_1^B\\ \beta_2^A\\ \beta_2^B\end{bmatrix}} + \underset{[2n,1]}{\begin{bmatrix}\zeta_1^A\\ \zeta_2^A\\ \zeta_1^B\\ \zeta_2^B\end{bmatrix}} \tag{8}$$

where $n = n_A + n_B$ is the number of elements in the model containing the two nodes and p_j is the number of explicative latent variables for each j-th endogenous construct $j = 1, \dots, J$ (in this example $J = 2$). We can define the design matrices X_0, with X corresponding to both hypotheses as:

$$X_0 = \underset{[2n,p_1+2p_2]}{\begin{bmatrix}X_1^A & 0 & 0\\ 0 & X_2^A & 0\\ X_1^B & 0 & 0\\ 0 & 0 & X_2^B\end{bmatrix}} \quad X = \underset{[2n,2p_1+2p_2]}{\begin{bmatrix}X_1^A & 0 & 0 & 0\\ 0 & X_2^A & 0 & 0\\ 0 & 0 & X_1^B & 0\\ 0 & 0 & 0 & X_2^B\end{bmatrix}} \tag{9}$$

Then we can write that $X_0 = XA$, defining matrix A as:

$$A = \underset{[2p_1+2p_2,p_1+2p_2]}{\begin{bmatrix}I_{p_1} & 0 & 0\\ 0 & I_{p_2} & 0\\ I_{p_1} & 0 & 0\\ 0 & 0 & I_{p_2}\end{bmatrix}} \tag{10}$$

where I_{P_j} is the identity matrix of order p_j. Then, according to **lemma 2** of Lebart [10], we can test the H_0 hypothesis by computing the following F statistic with (p_1) and $2(n - p_1 - p_2)$ degrees of freedom.

$$F_{Block} = \frac{(SS_{H_0} - SS_{H_1}) \Big/ p_1}{SS_{H_1} \Big/ 2(n - p_1 - p_2)} \tag{11}$$

5.2 F-coefficient test

Let us now suppose that the difference between the first structural equation in segment A and that in segment B is significant. We thus wish to identify the path coefficients responsible for this difference. Let us consider the same structural model shown in Figure 7. For the sake of simplicity, we wish to test the equality of coefficient β_1 in the first equation of both segments. We readapt the same F-global test to this situation. The two hypotheses are now written as follows:

$$H_0: \underset{[2n,1]}{\begin{bmatrix}\eta_1^A\\ \eta_2^A\\ \eta_1^B\\ \eta_2^B\end{bmatrix}} = \underset{[2n,2\sum_{j=1}^P p_j-1]}{\begin{bmatrix}\xi_1^A & 0 & 0 & 0 & 0\\ 0 & \xi_1^A & \eta_1^A & 0 & 0\\ \xi_1^B & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \xi_1^B & \eta_1^B\end{bmatrix}} \underset{[2\sum_{j=1}^P p_j-1,1]}{\begin{bmatrix}\beta_1\\ \beta_2^A\\ \beta_3^A\\ \vdots\\ \beta_3^B\end{bmatrix}} + \underset{[2n,1]}{\begin{bmatrix}\zeta_1^A\\ \zeta_2^A\\ \zeta_1^B\\ \zeta_2^B\end{bmatrix}} \tag{12}$$

$$H_1: \underset{[2n,1]}{\begin{bmatrix}\eta_1^A\\ \eta_2^A\\ \eta_1^B\\ \eta_2^B\end{bmatrix}} = \underset{[2n,2\sum_{j=1}^P p_j]}{\begin{bmatrix}\xi_1^A & 0 & 0 & 0 & 0 & 0\\ 0 & \xi_1^A & \eta_1^A & 0 & 0 & 0\\ 0 & 0 & 0 & \xi_1^B & 0 & 0\\ 0 & 0 & 0 & 0 & \xi_1^B & \eta_1^B\end{bmatrix}} \underset{[2\sum_{j=1}^P p_j,1]}{\begin{bmatrix}\beta_1^A\\ \beta_2^A\\ \beta_3^A\\ \vdots\\ \beta_3^B\end{bmatrix}} + \underset{[2n,1]}{\begin{bmatrix}\zeta_1^A\\ \zeta_2^A\\ \zeta_1^B\\ \zeta_2^B\end{bmatrix}} \tag{13}$$

Repeating the same rationale, of calling X_0 the design matrix of the null hypothesis and X the design matrix of the alternative hypothesis, we have $X_0 = XA$, where matrix A is now defined as follows:

$$A = \underset{[2\sum_{j=1}^P p_j,\sum_{j=1}^P p_j-1]}{\begin{bmatrix}1&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0\\0&0&0&1&0&0&0&0&0\\0&0&0&0&1&0&0&0&0\\1&0&0&0&0&0&0&0&0\\0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&1\end{bmatrix}} \tag{14}$$

Then, as before, we can test the H_0 hypothesis by computing the following F-coefficient statistic with 1 and $2(n-\sum_{j=1}^P p_j)$ degrees of freedom.

$$F_{Coefficient} = \frac{(SS_{H_0} - SS_{H_1}) \big/ 1}{SS_{H_1} \big/ 2(n-\sum_{j=1}^P p_j)} \tag{15}$$

6 Simulation study

In order to evaluate the sensitivity of the split criterion used in the F-global, F-block, and F-coefficient tests, we run a series of Monte Carlo simulations. Our simulations are based mainly on the works of [17], [18], and [19]. We have varied the data generating conditions: the sample size, the path coefficients for each segment, and the disturbance term for the endogenous construct. We have tried to reproduce similar conditions to those encountered in real-life applications of customer satisfaction studies.

6.1 Simulated models

The first step of our simulation study is the definition of the PLS model. In our model, data were generated according to the structural model in Figure 6, which we performed by following a two-step procedure [20]: first, we generated the latent variable data by following the relationship specified in the structural model; then, we reproduced the manifest variable data from the latent variables.

The PLS model consists of one exogenous (ξ) and two endogenous (η_1 and η_2) latent variables. The inner structure is defined as:

$$\begin{aligned} \eta_1 &= \beta_{\xi\eta_1}\xi + \zeta_{\eta_1} \\ \eta_2 &= \beta_{\xi\eta_2}\xi + \beta_{\eta_1\eta_2}\eta_1 + \zeta_{\eta_2} \end{aligned} \tag{16}$$

where β are regression coefficients and ζ are the error terms associated to the endogenous latent variables. The manifest variables are denoted by x for ξ and by y for η. The measurement models, for ξ, η_1 and η_2 are reflective and defined as:

$$\begin{aligned} x_1 &= \lambda_{x_1}\xi_1 + \varepsilon_{x_1} & y_1 &= \lambda_{y_1}\eta_1 + \varepsilon_{y_1} & y_4 &= \lambda_{y_4}\eta_2 + \varepsilon_{y_4} \\ x_2 &= \lambda_{x_2}\xi_1 + \varepsilon_{x_2} & y_2 &= \lambda_{y_2}\eta_1 + \varepsilon_{y_2} & y_5 &= \lambda_{y_5}\eta_2 + \varepsilon_{y_5} \\ x_3 &= \lambda_{x_3}\xi_1 + \varepsilon_{x_3} & y_2 &= \lambda_{y_3}\eta_1 + \varepsilon_{y_3} & y_6 &= \lambda_{y_6}\eta_2 + \varepsilon_{y_6} \end{aligned} \tag{17}$$

The λ terms are coefficients, and the ε terms are random errors. We set the λ values for the three constructs equal to 0.85, 0.80, 0.75 for $i = 1, \cdots, 3$ respectively. We used a Beta distribution $B \sim (6, 3)$ for the exogenous latent variable ξ, in order to reproduce the asymmetry that characterizes the way respondents answer in satisfaction studies.

6.2 Experimental factors

The levels of all factors in the experimental design were selected according to the following conditions:

1. *Size*. We consider three sample sizes as the total number of cases: {100, 400, and 1000}.

2. *Standard deviation of measurement errors*. We assume that the error terms, ε, follow a Normal distribution with zero expectation and three levels of standard deviation: small noise ($\sigma = 0.05$), moderate noise ($\sigma = 0.2$) and high noise ($\sigma = 1$).

3. *Difference between coefficients*. The model was estimated in two segments, A and B, while varying the level of difference between path coefficients (Figure 7): that is, they could be *equal* in both segments, or the difference could be *small*, *medium* or *large*. This means that we have a difference of 0, 0.2, 0.4 and 0.6, respectively, for the corresponding path coefficients of segments while always respecting the admissibility conditions of path coefficients.

In total, we have $3 \times 3 \times 4 = 36$ scenarios, which are the number of possible combinations of sample sizes, noise levels and differences between coefficients. We ran 50 repetitions for each experimental condition, and we present the mean of all 50 repetitions as an aggregate result.

6.3 Simulation study results

The results of the simulation study are graphically illustrated in Figure 8. We present only the marginal effects of the three experimental factors (sample sizes, noise levels, and differences between coefficients) which were used in the three compared F-tests (F-global, F-block, and F-coefficient test). In the nine plots, we graph from left to right first the F-global's p-values, second the F-block and, finally, the F-coefficient's p-values. This allows us to obtain a direct comparison of the three tests. We use a black circle to indicate the F-global's p-values, a blue circle and red triangle to differentiate between the F-block's p-values from the first equation and the p-values from the second equation, and a blue circle, red triangle and green plus sign to differentiate the F-coefficient's p-values for path coefficients β_1, β_2 and β_3. For the sake of interpretation, we have included in each plot the LOWESS regression line of the p-values as they relate to the evaluated experimental condition. In red, we show the LOWESS line representing the F-global trend. Again, to illustrate the F-block and F-coefficient trends, we use the same colors for the p-value points in order to differentiate between the p-value trend of the first latent equation and that of the second, and also to differentiate the p-values of the first path coefficient from the second and from the third. In the first three plots, it is possible to observe how the p-values decrease (i.e., they become more significant) as the sample size increases. In the second set of plots, we can see that sensitivity becomes lower when the level of noise for the error term grows higher. Finally, the influence of the difference between the path coefficients can be clearly appreciated in the last three plots, in which the p-values decrease as the difference between path coefficients increases. Hence, we can conclude that:

1. There is a clear effect of sample size: the larger the sample size is, the more sensitive the tests are.
2. There is a clear effect of the level of noise: the greater the level of noise is, the less sensitive the tests are.
3. There is a clear effect of the difference in the path coefficients in the two segments: the more different the path coefficients are, the more sensitive the tests are.

Finally, we present the results obtained in the simulation by varying only the β_1 coefficient (hence the η_1 equation) while keeping the other two coefficients constant. In Figure 9, we can see that the F-global detects the difference, as well as the F-block of η_1 and the F-coefficient of β_1. However, as expected, this does not occur with the F-block for η_2 and the F-coefficient for β_2 and β_3.

7 F-block and F-coefficient results for the split partition identified by Pathmox for the Spanish bank customers

At this point, it is interesting to use the F-block and the F-coefficient statistics to verify the improvement in the results, specifically in terms of our interpretation of the identified split between the "non-graduate" customers and the "graduate" customers.

In Tables 9 and 10, we present the results of the F-block and the F-coefficient tests, which are obtained by the R package "genpathmox" [16]. Starting with Table 9, we can see that the F-block identifies the equation of *Customer Satisfaction* as responsible for the split between the "non-graduate" customers and the "graduate" customers, with a p-value of *0.010*.

Regarding the path coefficients responsible for the difference, we can observe in Table 10 that the F-coefficient identifies the effect of *Perceived Quality* on *Customer Satisfaction* (F-statistic 7.514 and p-value: 0.006) and of *Perceived Value* on *Customer Satisfaction* (F-statistic 4.028 and p-value: 0.045).

Finally, we can conclude that the difference between the models of nodes 2 and 3 depends on the effects of two path coefficients: *Perceived Quality* on *Customer Satisfaction* and *Perceived Value* on *Customer Satisfaction*. Furthermore, as we expected, these two path coefficients are the same ones that we detected as most different when comparing a posteriori the inner models of the terminal nodes in Figure 4.

8 Discussion of findings

We have succeeded in detecting heterogeneity while also being able to identify the relationship of the variables responsible for such differences. This represents an important achievement in the decision making process particularly when dealing with complex models as in the case of PLS-PM. The extended version of the Pathmox approach does a step further by going deeper into the analysis of heterogeneity. The F-block and F-coefficient tests overcome an important limitation in the F-global test by revealing the reason for the obtained partitions. Pathmox currently offers two practical tools for identifying both the equations and the path coefficients that cause differences between two specific partitions. We tested the relative coherence between the F-block and the F-coefficient statistics in respect to the F-global, specifically when two path models are compared while taking into account: different sample sizes, different error levels in terms of the standard deviation of the manifest variables, and the various levels of difference between the path coefficients. In all cases, we have obtained very similar results with respect to the simulation study presented in [4]. Finally, we have seen the suitability of the two statistics in a practical marketing application. However, some aspects of the algorithm discussed in [4] are not solved yet. As with the F-global, the F-block and the F-coefficients use a parametric test based on a classical parametric statistic: the F statistic. This supposes the normality assumption of the perturbation terms with equal variance in all endogenous constructs, even though the assumptions are rarely met in practice. Nevertheless the sensitivity of the F statistic is guaranteed by larger sample size, lower levels of random perturbations and clearer differences in the segments as shown by the simulations that were undertaken.

Pathmox focuses only on the problem of detecting the path coefficients that are responsible for a difference between PLS-PM models by adapting the measurement model to every segment. This leads to the problem of invariance case that greatly increases in importance when we analyze data by considering potential sources of heterogeneity and fitting one model for every segment. In this situation, it becomes difficult to guarantee that each construct in each segment is measuring the same latent concept, and it is not reliable to compare the latent variables for individuals belonging to different segments. This is an important issue which we shall continue to investigate [24], and which deserves further research.

9 Conclusions

In this paper, we have extended the performance of the Pathmox approach in order to detect the equations (F-block test) of a structural model and the path coefficients (F-coefficient test) responsible for an observed difference. We have seen how the F-test of the model comparison made by Lebart ([10] and [11]) can be adapted to these purposes.

In the performed simulation, we have illustrated that the three tests mentioned above(F-coefficient, F-block and F-global) produce very consistent results and serve to detect the coefficients responsible for a split under the different experimental conditions.

Furthermore, we have shown the advantages of using the F-block and F-coefficient tests in real data applications.

The results revealed the existence of a niche of customers with specific drivers for *Customer Satisfaction*: "graduate" customers, for whom *Customer Satisfaction* is affected positively only as a result of the *Perceived Value* that is obtained. These findings can be explained as a consequence of graduates having greater financial knowledge. In contrast, for the majority of customers (the "non-graduate" customers) *Customer Satisfaction* is a holistic construct affected by all its drivers. Regarding *Loyalty*, no significant differences were discovered between the two groups of customers. Hence, we consider the global model to be valid for *Loyalty*.

References

[1] Esposito Vinzi V., Trinchera L., Amato S. (2010) PLS Path Modeling: From Foundations to Recent Developments and Open Issues for Model Assessment and Improvement. In: V. Esposito Vinzi, W.W. Chin, W.W., J. Henseler and H. Wang (Eds.), Handbook of Partial Least Squares. Concepts, Methods and Applications. Springer: Berlin, Germany;47-82.

[2] Bagozzi, R.,P., Youjae, Y. (2012) Specification, evaluation, and interpretation of structural equation models. Journal of the Academy of Marketing Science, 40 (1), 8-34; doi: $10.1007/s11747-011-0278-x$

[3] Sarstedt, M., Henseler, J., Ringle, C. (2011) M. Multi-group Analysis in Partial Least Squares (PLS) Path Modeling: Alternative Methods and Empirical Results. Advances in International Marketing, 22, 195-218; doi: 10.1108/$S1474-7979$

[4] Lamberti, G., Aluja, T. B., and Sanchez, G. (2016) The Pathmox approach for PLS path modeling segmentation. Applied Stochastic Models in Business and Industry; doi: 10.1002/$asmb$.2168.

[5] Keil, M., Tan, B. C., Wei, K. K., Saarinen, T., Tuunainen, V., & Wassenaar, A. (2000) A Cross-cultural Study on Escalation of Commitment Behavior in Software Projects. Management Information Systems Quarterly;24(2), 299-325; doi: 10.2307/3250940

[6] Chin, W.W. Frequently Asked Questions - Partial Least squares PLS-Graph. Available from http://disc-nt.cba.uh.edu/chin/plsfaq/plsfaq.htm [2000]

[7] Chin, W.W. (2003) A Permutation Based Procedure for Multi-Group Comparison of PLS Models. In: Proceedings of the PLS'03 International Symposium.

[8] Chin, W. W., & Dibbern, J. (2010) An Introduction to a Permutation Based Procedure for Multi-Group PLS Analysis: Results of Tests of Differences on Simulated Data and a Cross Cultural Analysis of the Sourcing of Information System Services Between Germany and the USA. In: V. Esposito Vinzi, W.W. Chin, W.W., J. Henseler and H. Wang (Eds.), Handbook of Partial Least Squares. Concepts, Methods and Applications. Springer: Berlin, Germany;171-193.

[9] Henseler, J. (2007) A New and Simple Approach to Multi-Group Analysis in Partial Least Squares Path Modeling. In: H. Martens and T. Naes. (Eds.), In: Proceedings of the PLS'07 International Symposium, Matforsk, As, Norway; 104-107

[10] Lebart, L., Morineau, A., Fenelon J.P. Traitement des Donnes Statistiques. Dunod, 1979.

[11] Chow, G.C. (1960) Test of Equality between Sets of Coefficients in two Linear Regressions. Econometrica; 28(3): 591-605. doi: 10.2307/1910133

[12] Fornell, C. (1992) A National Customer Satisfaction Barometer: The Swedish Experience. Journal of Marketing, 56(1): 6-21; doi:10.2307/1252129

[13] Martensen, A., Gronholdt, L., and Kristensen, K. (2000) The drivers of customer satisfaction and loyalty: cross-industry findings from Denmark. Total Quality Management, 11(4/5&6): 544-553; doi: 10.1080/09544120050007878

[14] Chin, W.W. (2010) How to Write Up and Report PLS Analyses. In: V. Esposito Vinzi, W.W. Chin, W.W., J. Henseler and H. Wang (Eds.), Handbook of Partial Least Squares. Concepts, Methods and Applications. Springer: Berlin, Germany;655-690.

[15] Sánchez, G., R package plspm. http://cran.r-project.org/web/packages/plspm [2012].

[16] Lamberti, G., R package genPathmox. http://cran.r-project.org/web/packages/genPathmox [2014]

[17] Chin, W.W., Marcolin, B.L., Newsted, P.R. (2003) A Partial Least Squares Latent Variable Modeling Approach for Measuring Interaction Effects: Results from a Monte Carlo Simulation Study and an Electronic-Mail Emotion/Adoption Study. Information Systems Research; 14(2): 189-217; doi: 10.1287/*isre*.14.2.189.16018

[18] Cassel, C., Hackl P., Anders, H.W. (1999) Robustness of Partial Least Squares Method for Estimating Latent Variable Quality Structures. Journal of Applied Statistics; 26(4): 435-446. doi: 10.1080/09544120050135443

[19] Westlund, A.H., Cassel, C.M., Eklöf J., Hackl P. (2001) Structural Analysis and Measurement of Customer Perceptions, Assuming Measurement and Specifications Errors. Total Quality Management. 12(7& 8): 873-881. doi:10.1080/09544120100000010

[20] Reinartz, W.J., Echambadi, R., Chin, W.W. Generating Non-normal Data for Simulation of Structural Equation Models Using Mattson's Method. Multivariate Behavioral Research, 2002. 37(2): 227-244. doi:10.1207/*S*15327906*MBR*3702_03

[21] Andreasen, T.W., and Lindestad, B. (1998) The effects of corporate image in the formation of customer loyalty. Journal of Service Marketing, 1, 82-92; doi: 10.1177/109467059800100107

[22] Anderson, E., Fornell, C., and Lehmann, D.R. (1994) Customer Satisfaction, Market Share, and Profitability: Findings from Sweden. Journal of Marketing, 58(3): 53-66; doi: 10.2307/1252310

[23] Kristensen, K., Martensen, A., and Gronholdt, L. (1999) Measuring the impact of buying behaviour on customer satisfaction. Total Quality Management, 10(4& 5): 602-614; doi: 10.1080/0954412997587

[24] Lamberti, G., Banet, T. (2017) Invariance Test: Detecting Difference Between Latent Variables Structure in Partial Least Squares Path Modeling. International Journal of Statistics and Probability, 6(2); doi: 10.5539/*ijsp*.*v*6*n*2*p*54

Table 1: Description of the manifest variables for each of the latent constructs

Latent variables	**Description of Indicators**
Image	1) Bank's reputation
	2) Trustworthiness
	3) Bank's solidity
	4) Innovation and forward looking
	5) Bank's emphasis on public affairs
	6) Caring about the customers needs
Expectations	1) Providing products and services to meet the customer's needs
	2) Providing customer service
	3) Providing solutions to daily banking needs
	4) Expectations of overall quality
Perceived Quality	1) Reliable products and services
	2) Range of products and services
	3) Degree to which customer feels well informed
	4) Personal advice
	5) Customer service
	6) Overall rating of perceived quality
	7) Clarity and transparency of operations and transactions
Perceived Value	1) Beneficial services and products
	2) Valuable investments
	3) Quality relative to price
	4) Price relative to quality
Customer Satisfaction	1) Overall rating of satisfaction
	2) Fulfillment of expectations
	3) Rating the performance relative to customer's ideal bank
Loyalty	1) Propensity to choose the same bank if the customer had to choose again
	2) Propensity to switch to other banks if they offered better terms
	3) Customer's intention to recommend the bank to friends or colleagues

Table 2: Codification of segmentation variables according to their type of scale and levels

Name	Scale	N. levels	Levels description
Gender	binary	2	female, male
Age	ordinal	5	equal to less than 25, 26 - 35, 36 - 45, equal to more than 46
Education level	ordinal	5	unfinished, elementary, high school, undergraduate, graduate
Occupation	nominal	5	manager, semi-employed, not employed, freelance, retired

Table 3: Description of latent variables of Bank dataset

LV	Description
Image	This refers to the brand name and the kind of associations customers get from the product/brand/company [21]. It is expected that image will have a positive effect on customer satisfaction and loyalty. In addition, image is also expected to have a direct effect on the expectations.
Expectations	This is based not on the actual consumer experience but on the accumulated information about quality from outside sources, such as advertising, word of mouth, and general media [22].
Perceived Quality	This comprises product quality (hardware) and service quality (software/humanware). Perceived product quality is the evaluation of the consumer's recent experience of products, as well as of associated services like customer service, conditions of product display, range of services and products, etc. [23]. Perceived quality is expected to affect *Satisfaction*.
Perceived Value	This is the perceived level of product quality relative to the price paid i.e., the value for the money in terms of the customer's experience [22]. Perceived value is expected to have a direct impact on *Satisfaction* and to be positively affected by *Perceived quality*.
Customer Satisfaction	This is defined as an overall evaluation of a firm's post-purchase performance or utilization of a service [12].
Loyalty	This refers to the repurchase intention and price tolerance of customers. It is the ultimate dependent variable in the model and it is expected that the better *Image* and higher *Customer Satisfaction* should increase customer *Loyalty*.

Table 4: Different measures for assessing the unidimensionality of blocks of indicators

Constructs	Type	N.Indicators	Crombach's α	Dillon ρ	1-Eigenvalue	2-Eigenvalue
Image	Reflective	6	0.867	0.901	3.627	0.823
Expectations	Reflective	4	0.836	0.891	2.684	0.532
Quality	Reflective	7	0.896	0.919	4.324	0.610
Value	Reflective	4	0.852	0.901	2.778	0.603
Satisfaction	Reflective	3	0.881	0.927	2.425	0.385
Loyalty	Reflective	3	0.826	0.897	2.234	0.554

Table 5: Summary results per blocks

Constructs	Type	Measur. type	Number Indicators	R.square	Av. Communality	Av. Redundance	AVE
Image	Exogenous	Reflective	6	0.000	0.600	0.000	0.600
Expectations	Endogenous	Reflective	4	0.375	0.669	0.251	0.669
Quality	Endogenous	Reflective	7	0.534	0.617	0.329	0.617
Value	Endogenous	Reflective	4	0.496	0.686	0.340	0.686
Satisfaction	Endogenous	Reflective	3	0.647	0.808	0.523	0.808
Loyalty	Endogenous	Reflective	3	0.653	0.739	0.483	0.739

Table 6: Bootstrap confidence intervals for path coefficients (0.025 and 0.975 percentiles)

Paths	Original path value	Bootstrap mean	Bootstrap stand. error	Bootstrap perc. .025	Bootstrap perc. .975	Significance
Image -> Satisfaction	0.160	0.156	0.062	0.046	0.282	YES
Expectations -> Satisfaction	0.107	0.099	0.051	0.003	0.202	YES
Quality -> Satisfaction	0.227	0.244	0.069	0.135	0.376	YES
Value -> Satisfaction	0.415	0.406	0.056	0.302	0.508	YES
Image -> Loyalty	0.244	0.245	0.038	0.174	0.320	YES
Satisfaction -> Loyalty	0.621	0.621	0.037	0.557	0.687	YES

Table 7: Direct, indirect and total effects of the path relationship corresponding to the node2 model of "non-graduate" customers and node 3 model of "graduate" customers

	$Node_2$			$Node_3$		
Paths	direct	indirect	total	direct	indirect	total
Image -> Satisfaction	0.175	0.301	0.476	0.058	0.424	0.482
Expectations -> Satisfaction	0.056	0.433	0.489	0.323	0.423	0.746
Quality -> Satisfaction	0.294	0.210	0.504	-0.256	0.321	0.065
Value -> Satisfaction	0.375	0.000	0.375	0.764	0.000	0.764
Image -> Loyalty	0.265	0.285	0.550	0.172	0.342	0.514
Expectations -> Loyalty	0.000	0.293	0.293	0.000	0.530	0.530
Quality -> Loyalty	0.000	0.302	0.302	0.000	0.046	0.046
Value -> Loyalty	0.000	0.225	0.225	0.000	0.543	0.543
Satisfaction -> Loyalty	0.600	0.000	0.600	0.711	0.000	0.711

Table 8: Terminal node path coefficient validation: bootstrap confidence intervals for path coefficients (0.025 and 0.975 percentiles) of node2 model of "non-graduate" customers and node 3 model of "graduate" customers

	$Node_2$			$Node_3$		
Paths	perc. .025	perc. .975	Significance	perc. .025	perc. .975	Significance
Image -> Satisfaction	0.085	0.285	YES	-0.186	0.492	NO
Expectations -> Satisfaction	-0.025	0.130	NO	-0.083	0.684	NO
Quality -> Satisfaction	0.181	0.408	YES	-0.627	0.310	NO
Value -> Satisfaction	0.252	0.457	YES	0.092	1.061	YES
Image -> Loyalty	0.173	0.335	YES	-0.089	0.552	NO
Satisfaction -> Loyalty	0.524	0.673	YES	0.349	0.941	YES

Table 9: F-block results

Latent variables	F-statistic	p-value	Significance
Satisfaction	3.039	0.010	YES
Loyalty	1.253	0.289	NO

Table 10: F-coefficient results

Paths	F-statistic	p-value	Significance
int -> Expectation	0.083	0.773	NO
Image -> Satisfaction	0.590	0.442	NO
Expectations->Satisfaction	1.785	0.182	NO
Quality->Satisfaction	7.514	0.006	YES
Value -> Satisfaction	4.028	0.045	YES
int -> Loyalty	2.293	0.130	NO
Image -> Loyalty	0.695	0.405	NO
Satisfaction -> Loyalty	0.680	0.410	NO

Figure 1: Path diagram of the *Customer Satisfaction* model

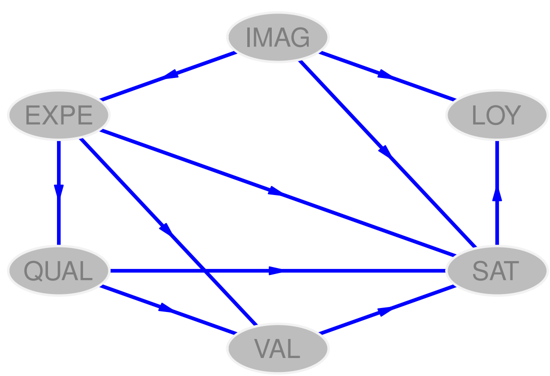

Figure 2: Estimated inner model of the Spanish bank

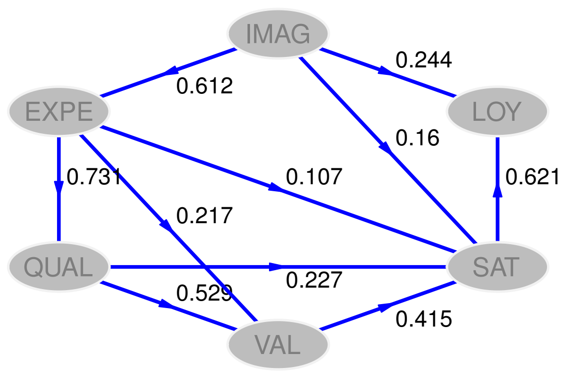

Figure 3: Pathmox tree of mobile data (from R genPathmox)

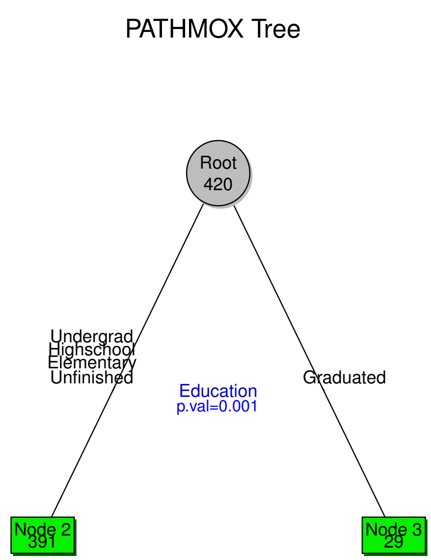

Figure 4: Estimated inner model of the terminal nodes: node2 (in green) model of "non-graduate" customers and node 3 (in yellow) model of "graduate" customers.

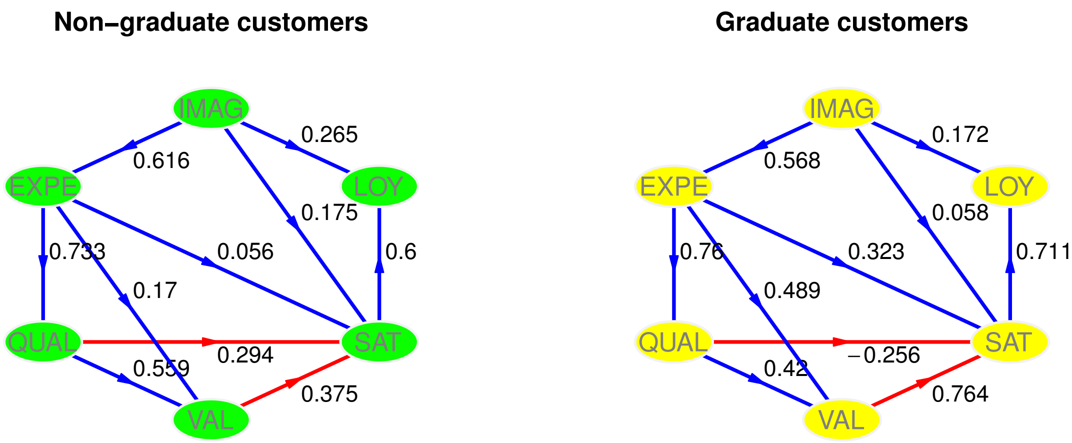

Figure 5: Path diagram of the structural model

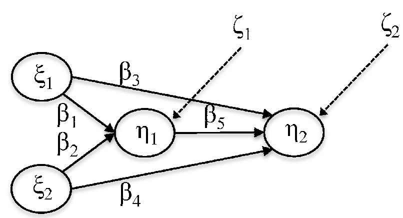

Figure 6: Path diagram of the simulation model

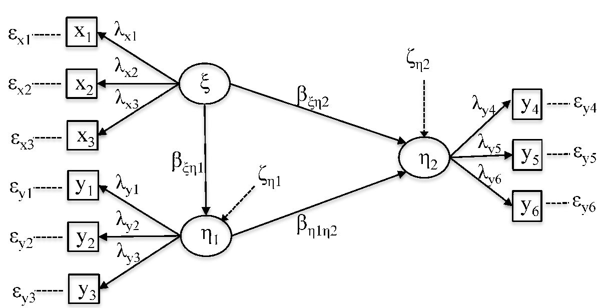

Figure 7: Comparison between the path coefficients of segments *A* and *B*

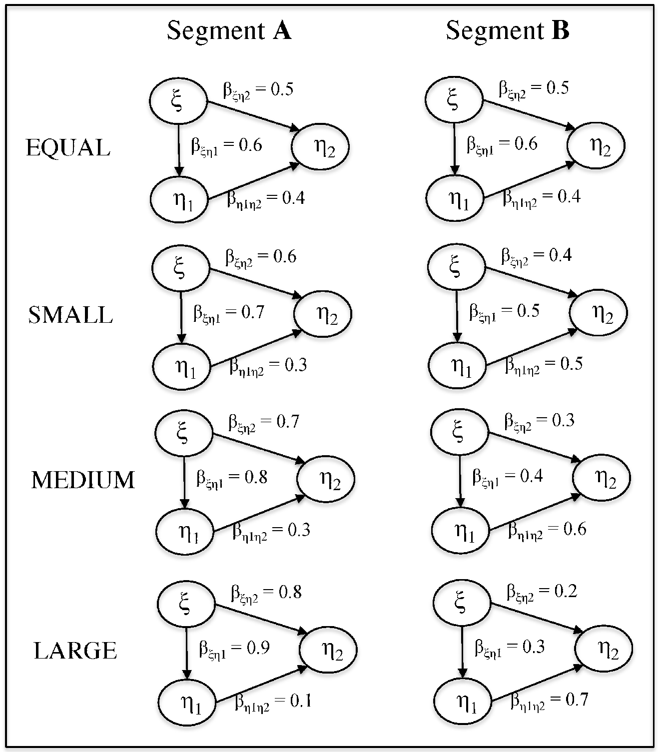

Figure 8: Comparison of F-block and F-coefficient statistics to the F-global by distinct simulation factors

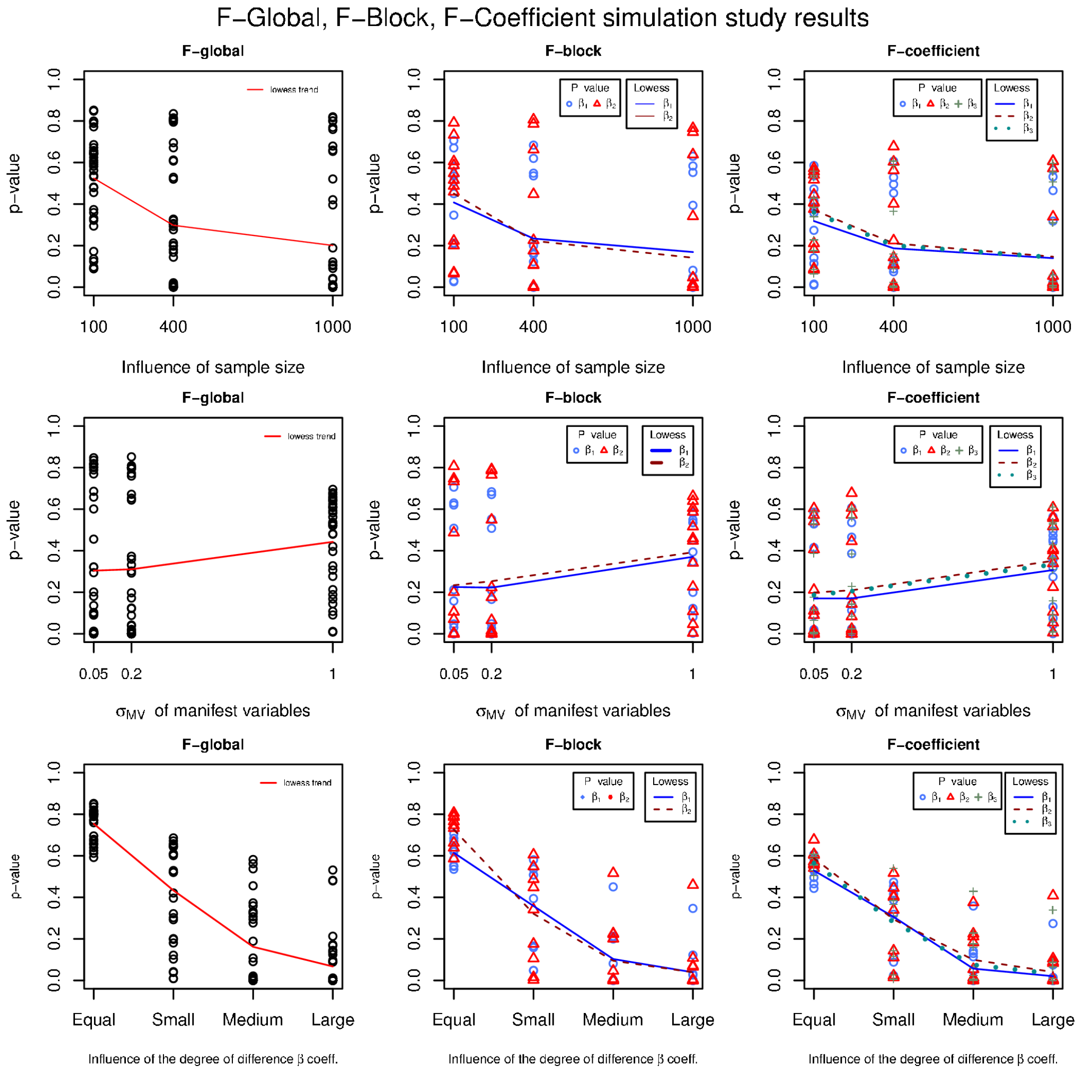

Figure 9: F-global, F-block and F-coefficient results, after varying only the β_1 coefficient